Data-Driven Highway Infrastructure Resilience Assessment

Final Report
August 2018

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16. Abstract

The transportation systems sector, one of the most critical infrastructure sectors in the US, has a subsector of highway and motor carrier industries that supports daily activities and emergency actions by providing services to other critical infrastructure segments such as healthcare and public health, emergency services, manufacturing, food and agriculture, etc. However, transportation networks face risks from natural and human-made events such as hurricanes, tsunamis, earthquakes, bridge collapse, and terrorist attacks. Thus, to improve the reliability of the components in interconnecting networks, it is necessary to consider these unpredictable failures in the network design. Resilient network design ensures that the network functionality is at an acceptable level of service in the presence of all probabilistic failures.

In this study, the authors addressed uncertainty in a transportation network by proposing a trilevel optimization model, which improves the resiliency of the network against uncertain disruptions. The link capacities are uncertain parameters and the origin-destination demands are deterministic. The goal was to minimize the total travel time under uncertain disruptions by designing a resilient transportation network. The trilevel optimization model has three levels. The lower level determines the network flow, the middle level assesses the resiliency of the network by identifying the worst-case scenario disruptions that could lead to a maximal travel time, and the upper level uses the system perspective to expand the existing transportation network to enhance the network's resiliency. In addition, the authors propose a new formulation for the network flow problem that will significantly reduce the number the number of variables and constraints.

The results of solving the trilevel optimization model can improve the resiliency of the network. However, this study was subject to some limitations, which suggested future research directions. In reality, transportation demands are not consistent, but the proposed model considers origin-destination demands as deterministic parameters. Relaxing this assumption requires a more complicated model to reflect uncertain demands. Other possible future work would be designing an exact algorithm to find the optimal solution.

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EXECUTIVE SUMMARY

The transportation systems sector, one of the most critical US infrastructure sectors, has a subsector of highway and motor carrier industries that supports daily activities and emergency actions by providing services to other critical infrastructure segments such as healthcare and public health, emergency services, manufacturing, food and agriculture, etc. However, transportation networks face risks from natural and human-made events such as hurricanes, tsunamis, earthquakes, bridge collapse, and terrorist attacks. Thus, to improve reliability in the components of interconnecting networks, it is necessary to consider these unpredictable failures in the network design. Resilient network design ensures that network functionality is at an acceptable level of service in the presence of all probabilistic failures.

In this study, the authors addressed uncertainty in a transportation network by proposing a trilevel optimization model, which improves the resiliency of the network against uncertain disruptions. The link capacities are uncertain parameters and the origin-destination demands are deterministic. The goal was to minimize the total travel time under uncertain disruptions by designing a resilient transportation network.

The trilevel optimization model has three levels. The lower level determines the network flow, the middle level assesses the resiliency of the network by identifying the worst-case scenario disruptions that could lead to a maximal travel time, and the upper level uses the system perspective to expand the existing transportation network to enhance the network's resiliency.

In addition, the authors proposed a new formulation for the network flow problem that would significantly reduce the number of variables and constraints.

The results of solving the trilevel optimization model can improve the network resiliency. However, this study was subject to some limitations that suggested future research directions. In reality, transportation demands are not consistent, but the proposed model considers origin-destination demands to be deterministic parameters. Relaxing this assumption requires a more complicated model to reflect uncertain demands. In addition, designing an exact algorithm to find the optimal solution poses an opportunity for future work.

INTRODUCTION

The transportation systems sector is one of the most critical US infrastructure sectors because many other critical infrastructure segments such as emergency services, food and agriculture, healthcare and public health, manufacturing, etc., depend on transportation networks to function properly. The highway and motor carrier industry is a subsector of transportation systems and supports the mobility of people, goods, and services, which are essential for daily activities and in emergency situations. However, road networks face risks from natural and human-made events such as hurricanes, tsunamis, earthquakes, bridge collapse, and terrorist attacks.

If there is an unstable design element in a network, the risky factor could cause more complex problems and has the power to disrupt a transportation network. This leads to a need to improve the reliability of components of interconnecting networks to guarantee safety and delivery of service in the presence of any unpredictable failures.

Resilient network design ensures that the network functionality remains at an acceptable level in the presence of all probabilistic failures.

The research team proposed adaptation of a trilevel optimization model (from their previous work published in Rahdar 2016 and Rahdar et al. 2018) for transportation network design, which will improve the resiliency of the network against uncertain disruptions. The middle and bottom levels depict the network interdiction problem, and the worst-case scenario disruptions that could lead to maximal travel times in the transportation system. The top level operates under the systems perspective, which relies on the optimal strategy to expand the existing transportation network so that it confronts the worst-case scenario disruptions in the most resilient manner possible.

The general mathematical model of the network design problem is a bilevel programming problem. The design and investment decisions are made at the upper level of the problem by decision-makers or system owners. The network flow decisions are made at the lower level of the problem by travelers. Some related pieces of research include LeBlanc and Boyce 1986, Marcotte 1986, Ben-Ayed et al. 1988, Migdalas 1995, Chiou 2005, and Zhang and Gao 2009.

Karoonsoontawong and Waller (2006) also proposed a linear bilevel programming model for the network design problem. They developed a genetic algorithm, simulated annealing, and a random search to solve the problem. Lin et al. (2011) formulated a bilevel linear program for the network design problem and proposed a heuristic algorithm based on Dantzig-Wolf decomposition to solve it; the solution of the algorithm could potentially be the local optimum. Farvaresh and Sepehri (2011) presented a single-level mixed integer linear programming formulation for bilevel design network problems. They also generated two valid inequalities to improve the efficiency of computation time. Khooban et al. (2015) proposed a bilevel programming model for the network design problem. The upper level problem of their model focused on expanding capacity and determining signal settings at intersections, and the lower level presented the assignment problem of user equilibrium.

Network design problems can be classified based on the origin-destination demand, the decision-making level, and the design variables. These problems are usually divided into two modeling cases in terms of demand: deterministic and uncertain demands. When the demand is deterministic, it is assumed that the demand between each origin-destination pair is given, but when it is uncertain, the origin-destination trip matrices are taken as random variables.

In addition, the network design problem involves making optimal decisions at three levels: strategic, tactical, and operational (Farahani et al. 2013, Chootinan et al. 2005). The strategic level includes long-term decisions such as building new links or expanding existing routes. Tactical decisions can determine the orientation of one-way roads or the allocation of lanes. Finally, the operational level decisions are short-term ones, involving traffic flow control and scheduling problems (Farahani et al. 2013).

Furthermore, network design problems can be classified into three groups based on the design variables. The first class is the discrete network design problem, which deals with adding a new lane or building a new road (Miandoabchi et al. 2012, Szeto et al. 2014, Miandoabchi et al. 2015). The second class is the continuous network design problem, which makes decisions on capacity enhancement, signal setting at intersections, and road pricing (Lo and Szeto 2003, Chiou 2005, Szeto and Lo 2008, Lo and Szeto 2009). The third class involves both discrete and continuous design variables (Cantarella et al. 2006, Zhang and Gao 2009, Gallo et al. 2010).

This project studied the strategic decisions about link capacity expansions by adding new lanes to critical links. To identify which link is critical, the authors proposed a trilevel optimization model to identify the vulnerable links and decide on expansions.

Identifying critical links of a transportation network is the main issue in vulnerability analysis because failure of these links has a huge impact on the whole network. Some researchers (Jenelius et al. 2006, Taylor et al. 2006) assumed that failure is a link, or a group of links, being completely disrupted and examined the effect of iteratively removing road links to calculate the network performance. As stated by Bagloee et al. (2017), this may ignore the potential combined effects of multiple links.

For example, if there are two bridges on a river, one of them could be congested and the traffic would shift to the other one. Thus, it is possible neither of them is considered as a critical link individually, but if they are disrupted at the same time, they would become vulnerable links. In addition, this approach can be computationally intensive (Chen et al. 2012).

Another approach to identify the critical links is preselecting potentially vulnerable links by calculating the stochastic traffic assignment. Knoop et al. (2007) compared 10 different criteria for selecting potentially vulnerable links in a network and concluded that none of these strategies accurately predicted the list of vulnerable links. Also, they stated that combining the links did not present a true representation of the full consequences of blocking a link. Therefore, these strategies are not accurate enough to properly identify the critical links in a road network.

The aim of this project was to propose a model that minimizes the effects of total network disruption for a transportation network. In order to build a resilient transportation network that can manage uncertain disruptions, the authors worked on the uncertainty of link capacities and proposed a new approach to perform network design. They also assumed that the origin-destination demands are constant.

For example, a severe weather event could cut one lane while the number of travelers and the distance for each link are fixed. Since reducing the link capacities degrades the performance of a transportation network and can delay or stop movement, quantifying such impacts is critical to network design improvement.

The contributions of this project are as follows: First, the researchers developed a new trilevel optimization model for the resilient network design problem. Second, they proposed a new formulation for the network flow problem (the third level of the trilevel model) to reduce the number of variables and constraints significantly. Third, they designed a heuristic algorithm for solving the trilevel optimization model to efficiently assess and enhance the resiliency of the network.

MODEL FORMULATION

Problem Statement

The authors addressed a traffic network design problem related to uncertainty over available link capacity. The goal was to make decisions on how to strengthen the network against future possible disruptions under conditions of limited resources. The objective function was minimization of travel cost, subject to the network expansion budget and travel demand satisfaction.

Network design problems are usually formulated as a two-stage decision making problem where the link capacity expansion decisions are made at the first stage before the realization of disruption uncertainty, and flow variables are decided after observing the disruption in the second stage. In the remainder of the chapter, the authors first describe the network flow problem and then introduce the trilevel optimization model.

The Network Flow Problem

The minimum cost network flow model in the literature is represented in Equation 1 below. The objective of the model is to minimize the total travel cost while adding a penalty for unsatisfied travel demands. Table 1 includes the notation used in formulating the traditional network flow model.

Table 1. Notation used in the network flow model

Sets		
$\mathcal{N}$		Set of nodes
$\mathcal{L}$		Set of links
$\mathcal{T}$		Set of destination nodes
$\mathcal{S}(t)$		Set of source nodes with destination $t \in \mathcal{T}$
$\mathcal{D}$		Set of source-destination travel demands
Decision variables		
$g_{i,j,s,t}$	(Veh/hr)	Flow of the link $(i,j) \in \mathcal{L}$ for demand of $(s,t) \in \mathcal{D}$
$\delta_{s,t}$	(Veh/hr)	Unsatisfied travel demand of $(s,t) \in \mathcal{D}$
Parameters		
$\tau_{i,j}$	($/Veh/hr)	Travel cost of the link $(i,j) \in \mathcal{L}$
ρ	($/Veh/hr)	Penalty cost for left over travel demand
$d_{s,t}$	(Veh/hr)	Travel demand of $(s,t) \in \mathcal{D}$
$p_{i,j}$	(Ln)	Capacity of the link $(i,j) \in \mathcal{L}$

$$\min \quad \sum_{(i,j)\in\mathcal{L}} \sum_{(s,t)\in\mathcal{D}} \tau_{i,j}\, g_{i,j,s,t} + \rho \sum_{(s,t)\in\mathcal{D}} \delta_{s,t} \qquad \textbf{(1a)}$$

$$
\text{s.t.} \quad \sum_{(i,j)\in\mathcal{L}} g_{i,j,s,t} - \sum_{(i,j)\in\mathcal{L}} g_{j,i,s,t} = -d_{s,t} + \delta_{s,t} \qquad \forall t \in \mathcal{T}, s \in \mathcal{S}(t), i = t \quad \textbf{(1b)}
$$

$$
\sum_{(i,j)\in\mathcal{L}} g_{i,j,s,t} - \sum_{(i,j)\in\mathcal{L}} g_{j,i,s,t} = d_{s,t} - \delta_{s,t} \qquad \forall t \in \mathcal{T}, s \in \mathcal{S}(t), i = \mathcal{S}(t) \quad \textbf{(1c)}
$$

$$
\sum_{(i,j)\in\mathcal{L}} g_{i,j,s,t} - \sum_{(i,j)\in\mathcal{L}} g_{j,i,s,t} = 0 \qquad \forall t \in \mathcal{T}, s \in \mathcal{S}(t), i = \mathcal{N}\backslash\{t, \mathcal{S}(t)\} \quad \textbf{(1d)}
$$

$$
\sum_{(s,t)\in\mathcal{D}} g_{i,j,s,t} \leq 2000\, p_{i,j} \qquad \forall (i,j) \in \mathcal{L} \quad \textbf{(1e)}
$$

$$
g_{i,j,s,t} \geq 0 \qquad \forall (i,j) \in \mathcal{L}, (s,t) \in \mathcal{D} \quad \textbf{(1f)}
$$

$$
\delta_{s,t} \geq 0 \qquad (s,t) \in \mathcal{D} \quad \textbf{(1g)}
$$

One potential drawback of this formulation is that it generates a model with a large number of variables and constraints. The authors present an equivalent and more efficient model with fewer variables and constraints, shown in Equation 3. In the new formulation, the definition of variable $g_{i,j,s,t}$ is changed to $g'_{i,j,t}$, which is the flow of the link (i,j) to destination t.

$$
g'_{i,j,t} = \sum_{s\in\mathcal{S}(t)} g_{i,j,s,t} \qquad \textbf{(2)}
$$

Therefore, the reformulation of the network flow problem is as follows:

$$
\min \quad \sum_{(i,j)\in\mathcal{L}} \sum_{(s,t)\in\mathcal{D}} \tau_{i,j}\, g'_{i,j,t} + \rho \sum_{(s,t)\in\mathcal{D}} \delta_{s,t} \qquad \textbf{(3a)}
$$

$$
\text{s.t.} \quad \sum_{(j,t)\in\mathcal{L}} g'_{j,t,t} = \sum_{s\in\mathcal{S}(t)} d_{s,t} - \delta_{s,t} \qquad \forall t \in \mathcal{T} \quad \textbf{(3b)}
$$

$$
\sum_{(s,j)\in\mathcal{L}} g'_{s,j,t} - \sum_{(j,s)\in\mathcal{L}} g'_{j,s,t} = d_{s,t} - \delta_{s,t} \qquad \forall t \in \mathcal{T}, s \in \mathcal{S}(t) \quad \textbf{(3c)}
$$

$$
\sum_{(i,j)\in\mathcal{L}} g'_{i,j,t} - \sum_{(j,i)\in\mathcal{L}} g'_{j,i,t} = 0 \qquad \forall t \in \mathcal{T}, i = \mathcal{N}\backslash\{t, \mathcal{S}(t)\} \quad \textbf{(3d)}
$$

$$
\sum_{t\in\mathcal{T}} g'_{i,j,t} \leq 2000\, p_{i,j} \qquad \forall (i,j) \in \mathcal{L} \quad \textbf{(3e)}
$$

$$
g'_{i,j,t} \geq 0 \qquad \forall (i,j) \in \mathcal{L}, t \in \mathcal{T} \quad \textbf{(3f)}
$$

$$
\delta_{s,t} \geq 0 \qquad (s,t) \in \mathcal{D} \quad \textbf{(3g)}
$$

Constraint (3b) states that the total input flows with destination t to the node t should be equal to the total travel demand of node t minus leftover travel demand. Similarly, Constraint (3c) expresses that the total output flows of node s with destination t, minus the total input flows to node s with destination t, is equal to the travel demand from origin s to destination t, minus leftover travel demand. Constraint (3d) states that the input flows with destination t to the node i, in which there is no demand from i to t, equal the output flows of the node i with destination t. Constraint (3e) means that the flow of the link (i, j) for all destinations cannot exceed its capacity. It is assumed that each lane has the capacity of 2,000 vehicle per hour.

The actual objective function to calculate the total travel time is a nonlinear function. Thus, to be more realistic, the authors defined the concept of blocks for each link which enabled them to construct a piecewise linear approximation.

Each link can have several blocks; the capacity of each block on a link equals the capacity of the link, but the travel times are different. For example, a link has two lanes, the authors assign five blocks for this link, and each one has two lanes. The travel time of the first block is the regular travel time. The second block has a travel time twice as long as the first one, and so on. Therefore, if the number of vehicles traveling through the link is more than the capacity of the link, there is no unsatisfied demand, but the authors' calculations require a block with longer travel time. To do this, the authors needed a new variable, $f_{i,j,b}$, which is the flow of the link $(i, j) \in \mathcal{L}$ using block b capacity. The new reformulation appears in Equation 4.

$$\min \quad \sum_{(i,j)\in\mathcal{L}} \sum_{b} \tau_{i,j,b}\, f_{i,j,b} \tag{4a}$$

$$\text{s.t.} \quad \sum_{(j,t)\in\mathcal{L}} g_{j,t,t} = \sum_{s\in\mathcal{S}(t)} d_{s,t} \qquad \forall t \in \mathcal{T} \tag{4b}$$

$$\sum_{(s,j)\in\mathcal{L}} g_{s,j,t} - \sum_{(j,s)\in\mathcal{L}} g_{j,s,t} = d_{s,t} \qquad \forall t \in \mathcal{T}, s \in \mathcal{S}(t) \tag{4c}$$

$$\sum_{(i,j)\in\mathcal{L}} g_{i,j,t} - \sum_{(j,i)\in\mathcal{L}} g_{j,i,t} = 0 \qquad \forall t \in \mathcal{T}, i = \mathcal{N}\backslash\{t, \mathcal{S}(t)\} \tag{4d}$$

$$\sum_{b} f_{i,j,b} = \sum_{t\in\mathcal{T}} g_{i,j,t} \qquad \forall (i,j) \in \mathcal{L} \tag{4e}$$

$$f_{i,j,b} \leq 2000\, p_{i,j} \qquad \forall b, (i,j) \in \mathcal{L} \tag{4f}$$

$$g_{i,j,t} \geq 0 \qquad \forall (i,j) \in \mathcal{L}, t \in \mathcal{T} \tag*{(4e)}$$

Trilevel Optimization Model

Relaxing the simplifying assumption of no disruption in the network resulted in a two-stage decision-making problem in which uncertain disruptions occurred after the expansion decisions were made. Disruptions are uncertain variables in the problem, but the lower and upper bounds are known.

The authors proposed a two-stage trilevel optimization model to make decisions on expansion, identify the disruptions, and determine the network flow. The authors assumed that after decisions (expansion decisions) are made at the first stage, uncertain disruptions will be observable and, thus, the second stage (network flow problem) becomes a deterministic model.

To make expansion decisions in the first stage, the authors took a pessimistic view of uncertainty and anticipated the worst-case scenario for the second stage. Using this formulation, the authors designated three levels in the model. The first stage decisions are made in the upper level, the worst-case scenario is identified in the middle level (given the first stage decision), and the second stage decisions are made in the lower level under the worst-case scenario (given the first stage decision). The trilevel optimization model was developed using notations defined in Table 2.

Table 2. Notation in the trilevel model

Sets		
$\mathcal{N}$		Set of nodes
$\mathcal{L}$		Set of links
$\mathcal{T}$		Set of destination nodes
$\mathcal{S}(t)$		Set of source nodes with destination $t \in \mathcal{T}$
$\mathcal{D}$		Set of source-destination travel demands
Decision variables for the upper level $\boldsymbol{x}$		
$\pi_{i,j}$	(Ln)	Capacity expansion of the link $(i,j) \in \mathcal{L}$
Decision variables for the middle level $\boldsymbol{y}$		
$q_{i,j}$	(Ln)	Capacity reduction of the link $(i,j) \in \mathcal{L}$ because of disruption
Decision variables for the lower level $\boldsymbol{z}$		
$g_{i,j,t}$	(Veh/hr)	Flow of the link $(i,j) \in \mathcal{L}$ to destination $t \in \mathcal{T}$
$f_{i,j,b}$	(Veh/hr)	Flow of the link $(i,j) \in \mathcal{L}$ using block b capacity
Parameters		
$\tau_{i,j,b}$	($/Veh/hr)	Travel cost of the link $(i,j) \in \mathcal{L}$ using block b capacity
$a_{i,j}$	($/Ln)	Capacity expansion cost of the link $(i,j) \in \mathcal{L}$
$l^{\mathrm{x}}_{i,j}$	(Ln)	Lower bound of capacity expansion for the link $(i,j) \in \mathcal{L}$
$u^{\mathrm{x}}_{i,j}$	(Ln)	Upper bound of capacity expansion for the link $(i,j) \in \mathcal{L}$
B	($)	Network expansion budget
Q	(Ln)	Network disruption upper bound
$l^{\mathrm{y}}_{i,j}$	(Ln)	Lower bound of disruption for the link $(i,j) \in \mathcal{L}$
$u^{\mathrm{y}}_{i,j}$	(Ln)	Upper bound of disruption for the link $(i,j) \in \mathcal{L}$
$d_{s,t}$	(Veh/hr)	Travel demand of $(s,t) \in \mathcal{D}$
$p_{i,j}$	(Ln)	Capacity of the link $(i,j) \in \mathcal{L}$

To better illustrate the trilevel model, the authors aggregated the decision variables of three levels into x, y, and z, respectively, and aggregated objective function coefficients into c. The objective function is represented in (4a). Using the notations of aggregated decision variables and parameters, the authors formulated the trilevel optimization model as follows.

$$\min_{x \in \mathcal{X}} \left\{ \max_{y \in \mathcal{Y}(x)} \left\{ \min_{z \in \mathcal{Z}(x,y)} c^\top z \right\} \right\} \tag{5}$$

Here, the lower level solves a deterministic problem, $\min_{z \in \mathcal{Z}(x,y)} c^\top z$, to minimize the total travel cost given the expansion decision, x, made at the upper level and the worst-case scenario of disruptions, y, identified by the middle level. The feasible set $\mathcal{Z}(x,y)$ is defined in Equation 6 and its compact form is represented in Equation 7 where A_3^{x}, A_3^{y}, and A_3^{z} are coefficients of the upper, middle, and lower level variables, respectively.

$$\mathcal{Z}(x,y) = \left\{ \begin{array}{lll} z: & \sum_{(j,t)\in\mathcal{L}} g_{j,t,t} = \sum_{s\in\mathcal{S}(t)} d_{s,t} & \forall t \in \mathcal{T} \quad \text{(6a)} \\ & \sum_{(s,j)\in\mathcal{L}} g_{s,j,t} - \sum_{(j,s)\in\mathcal{L}} g_{j,s,t} = d_{s,t} & \forall t \in \mathcal{T}, s \in \mathcal{S}(t) \quad \text{(6b)} \\ & \sum_{(i,j)\in\mathcal{L}} g_{i,j,t} - \sum_{(j,i)\in\mathcal{L}} g_{j,i,t} = 0 & \forall t \in \mathcal{T}, i = \mathcal{N}\backslash\{t, \mathcal{S}(t)\} \quad \text{(6c)} \\ & \sum_{b} f_{i,j,b} = \sum_{t\in\mathcal{T}} g_{i,j,t} & \forall (i,j) \in \mathcal{L} \quad \text{(6d)} \\ & f_{i,j,b} \leq 2000(p_{i,j} + \pi_{i,j} - q_{i,j}) & \forall b, (i,j) \in \mathcal{L} \quad \text{(6e)} \\ & g_{i,j,t} \geq 0 & \forall (i,j) \in \mathcal{L}, t \in \mathcal{T} \quad \text{(6f)} \end{array} \right\}$$

$$A_3^{\mathrm{x}}\, x + A_3^{\mathrm{y}}\, y + A_3^{\mathrm{z}}\, z \leq b_3 \tag{7}$$

The middle level observes the expansion decision, x, made at the upper level and solves a bilevel optimization model, $\max_{y \in \mathcal{Y}(x)} \left\{ \min_{z \in \mathcal{Z}(x,y)} c^\top z \right\}$, to identify the worst-case scenario of disruptions, anticipating the response of the lower level. The feasible set $\mathcal{Y}(x)$ is defined in Equation 8 and its compact form is represented in Equation 9 where A_2^{y} is the coefficient of the middle level variables. The first constraint is the bound on the total disruption in the network and the second constraint set the lower and upper bounds for the disruption of each link (i,j) in the network.

$$\mathcal{Y}(x) = \left\{ \begin{array}{lll} y: & \sum_{(i,j)\in\mathcal{L}} q_{i,j} \leq Q & \\ & l_{i,j}^{\mathrm{y}} \leq q_{i,j} \leq u_{i,j}^{\mathrm{y}} & \forall (i,j) \in \mathcal{L} \\ & q_{i,j} \in \mathbb{Z} & \forall (i,j) \in \mathcal{L} \end{array} \right\} \tag{8}$$

$$A_2^{\mathrm{y}}\, y \leq b_2 \tag{9}$$

The upper level solves the trilevel optimization model (Equation 5), which minimizes the travel cost, anticipating the response from the middle and lower levels. The feasible set $\mathcal{X}$ is defined in

Equation 10 and its compact form is represented in Equation 11 where A_1^{x} is the coefficient of the upper level variables. The first constraint is the budget limit on the total expansion in the network and the second constraint set the lower and upper bounds for the expansion of each link (i, j) in the network.

$$\mathcal{X} = \left\{ \begin{array}{lll} x: & \sum_{(i,j)\in\mathcal{L}} a_{i,j}\pi_{i,j} \le B & \\ & l_{i,j}^{\mathrm{x}} \le \pi_{i,j} \le u_{i,j}^{\mathrm{x}} & \forall (i,j) \in \mathcal{L} \\ & \pi_{i,j} \in \mathbb{Z} & \forall (i,j) \in \mathcal{L} \end{array} \right\} \quad \mathbf{(10)}$$

$$A_1^x\, x \le b_1 \quad \mathbf{(11)}$$

ALGORITHM DESIGN

The upper level determines the capacity expansion, the middle level identifies the worst-case scenario for disruptions given the expansion decisions, and the lower level makes the flow decisions given the expansions and disruptions. The authors deconstruct the trilevel model into a master problem and a subproblem. To develop the master problem, the authors formulated the dual of the lower level problem. The compact form of the lower level problem is shown in Equation 12 when expansions and disruptions are given.

$$\min \quad c^{\top} z \qquad \textbf{(12a)}$$
$$\text{s.t.} \quad A_3^{\mathrm{z}}\, z \leq b_3 - A_3^{\mathrm{x}}\, \hat{x} - A_3^{\mathrm{y}}\, \hat{y} \qquad \textbf{(12b)}$$

The dual of the lower level problem is represented in Equation 13, where λ is the dual variable.

$$\max \quad -\left(b_3 - A_3^{\mathrm{x}}\, \hat{x} - A_3^{\mathrm{y}}\, \hat{y}\right)^{\top} \lambda \qquad \textbf{(13a)}$$
$$\text{s.t.} \quad -A_3^{\mathrm{z}\,\top}\, \lambda \leq c \qquad \textbf{(13b)}$$

The master problem $M^{\mathrm{tri}}(\hat{y}, \hat{\lambda})$ is formulated in Equation 14. It consists of the upper level problem constraints (14b) and a series of optimality cuts (14c) which are added to the master problem in each iteration. It has two decision variables: variable x which is the expansion decision, and variable t_M which is the total travel time. Parameters $\hat{\lambda}_s$ and $\hat{y}_s$ are, respectively, the dual variable values of the lower level problem and the disruption amount, which are both estimated by solving the subproblem in iteration s.

$$\min \quad t_M \qquad \textbf{(14a)}$$
$$\text{s.t.} \quad A_1^{x}\, x \leq b_1 \qquad \textbf{(14b)}$$
$$t_M \geq -\left(b_3 - A_3^{\mathrm{x}}\, x - A_3^{\mathrm{y}}\, \hat{y}_s\right)^{\top} \hat{\lambda}_s \qquad \forall s \qquad \textbf{(14c)}$$

The subproblem $S^{\mathrm{tri}}(\hat{x})$, represented in Equation 15, is a bilevel (middle and lower levels) optimization problem to assess the resiliency and determine the network flow. It has two variables: variable y which is the disruption in the network and z which is the network flow. The parameter $\hat{x}$ is the expansion decision made in the master problem.

$$\max_{y \in \mathcal{Y}(\hat{x})} \left\{ \min_{z \in Z(\hat{x}, y)} c^{\top} z \right\} \qquad \textbf{(15)}$$

The idea was to iteratively solve the master problem $M^{\mathrm{tri}}(\hat{y}, \hat{\lambda})$ and the subproblem $S^{\mathrm{tri}}(\hat{x})$ to determine capacity expansions under the worst case of disruption. In the first iteration, the authors assumed that there is no expansion for links and solved the subproblem (Equation 15) to assess the resiliency of the network and determine the network flow. Then, with the given disruption and network flow, the authors solved the master problem (Equation 14) to expand the critical links. By adding the new link capacities, the authors solved the subproblem (Equation 15) again to find the network flow under the worst-case disruption. The resulting bilevel model (Equation 15) will either confirm the solution of the upper level decision so the algorithm terminates, or yield a worst-case scenario that will be added to the master problem as a new cut in the next iteration.

The subproblem (Equation 15) is a bilevel programming problem that needs an algorithm to be solved. Therefore, the authors also designed a heuristic algorithm to solve the subproblem (Equation 15) with fixed upper level decisions x from the solution of the master problem (Equation 14).

In the first iteration, the authors solved the lower level problem (Equation 12) assuming that there is no disruption in the network. Then, with the resulting data from the lower level problem, the authors determined a worse disruption scenario. To find a new worse scenario, the authors needed the dual variables λ for the capacity constraints of the lower level problem (6e).

In the first iteration, the authors cut as many links with the largest dual variables as the disruption budget Q allows. With the new disruption, the lower level problem was solved again to find the network flow under the new scenario.

In the next iterations with the new dual variables, the authors improved the previous solution by cutting and uncutting one lane in each iteration. The authors cut a new lane with the largest dual variable and uncut one disrupted lane with the smallest dual variable in each iteration. This procedure continued until there was no improvement in the objective function of the problem (Equation 15).

The algorithm flowchart is shown in Figure 1 and the steps of the algorithm are described in Algorithm 1.

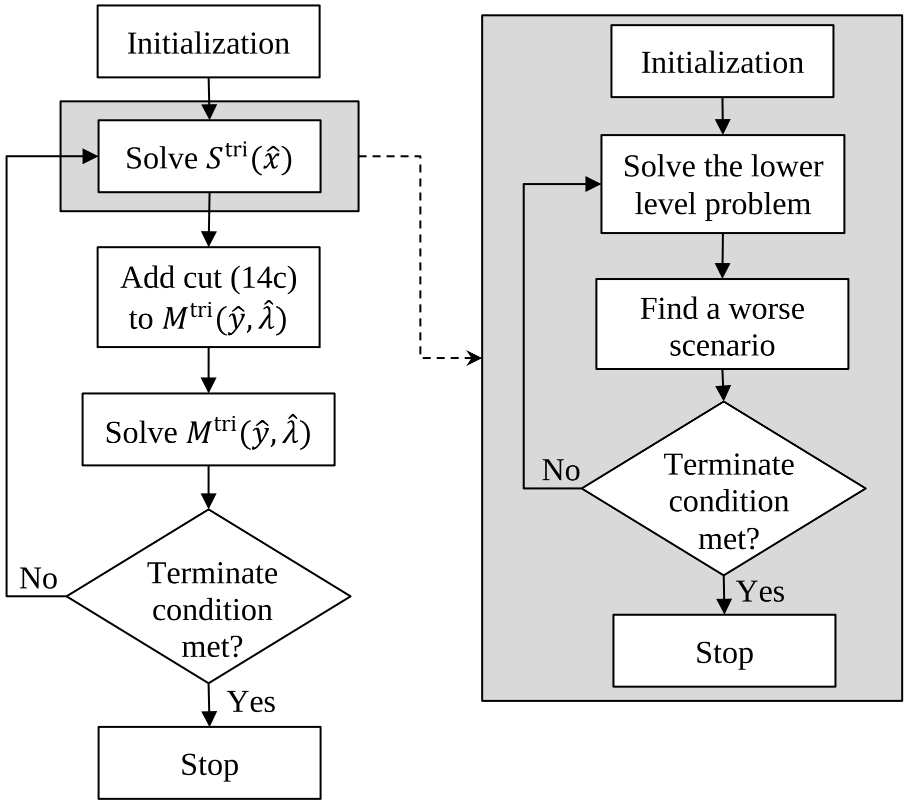


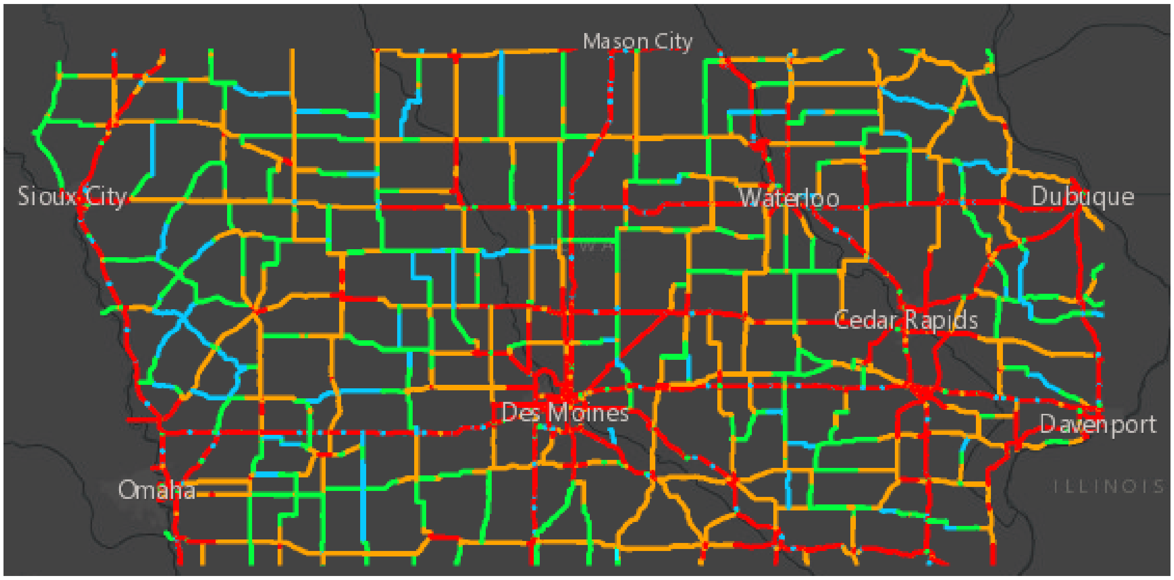
Figure 1. The algorithm flowchart

Algorithm 1. The algorithm of solving the trilevel model (Equation 5)

Inputs: $\mathcal{X}$, $\mathcal{Y}(x)$, and $\mathcal{Z}(x, y)$, $\forall x \in \mathcal{X}, y \in \mathcal{Y}$
Initialize $(x^*, y^*, z^*) = \emptyset$, $\hat{x} = 0$, $\zeta^L = -\infty$, $\zeta^U = +\infty$
while $\zeta^L < \zeta^U$ **do**
    Solve subproblem $S^{\text{tri}}(\hat{x})$ as follows
        Initialize $\hat{y} = 0$, $\zeta = -\infty$.
        while *there is an improvement* **do**
            Solve the lower level problem (12). Let $\hat{z}$ denote the network flow and $\hat{\lambda}$ denote the dual variable of link capacity constraints. Update $\zeta \leftarrow c^\top \hat{z}$.
            Find a worse scenario. Let $\hat{y}$ denote the disruption in the network.
        end while
    Return $\hat{y}$, $\hat{z}$, and $\hat{\lambda}$. Update $\zeta^U \leftarrow \min\{\zeta^U, c^\top \hat{z}\}$.
    Add cut (14c) to master problem $M^{\text{tri}}(\hat{y}, \hat{\lambda})$.
    Solve master problem $M^{\text{tri}}(\hat{y}, \hat{\lambda})$.
    Let $\hat{x}$ and t_M denote an optimal solution. Update $\zeta^L \leftarrow t_M$.
end while
Return $x^* = \hat{x},\ y^* = \hat{y},\ z^* = \hat{z}$.

COMPUTATIONAL EXPERIMENTS

To demonstrate the model, the authors applied it to the Iowa transportation network as shown in Figure 2.



Iowa Traffic Data, Iowa DOT 2018

Figure 2. Iowa transportation network

To conduct the numerical experiment, they simplified the network with 70 nodes, 224 links, and 924 origin-destination (OD) pairs, which are represented in Figure 3.

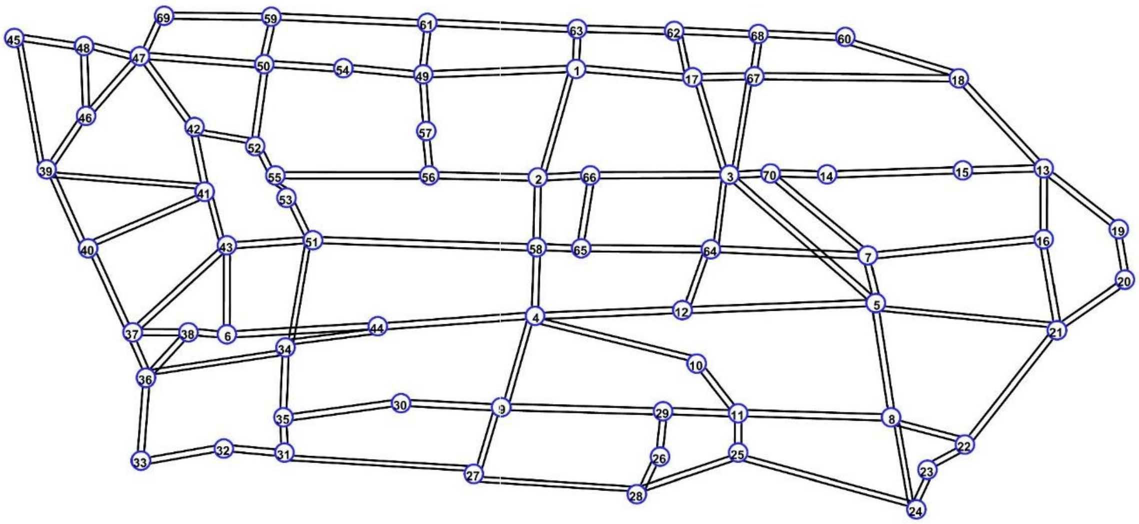


Figure 3. Simplified Iowa transportation network

The distance between two locations is defined as the Euclidian distance and the authors used the gravity model to estimate the demand of each OD pair. To apply the gravity model, the authors needed the friction and socioeconomic factors and the production/attraction matrix. The friction factor is calculated from the gamma function, $F_{ij} = at_{ij}^{b}e^{ct_{ij}}$, where t_{ij} is the travel time between node i and node j. In this model, the trips between OD pairs are home-based work (HBW); thus, the parameters of the gamma function from Martin and McGuckin (1998) are: $a = 28507$, $b = -0.02$, and $c = -0.123$. The authors also assumed that the socioeconomic factor was K_{ij} = 1 for all links.

To estimate the production/attraction of each node, the authors used the annual average daily traffic (AADT) of the Iowa transportation network. The authors applied a peak hour factor of 12% and assumed a 50% to 50% split of two-directional flow. Therefore, they used 6% as the inflow and 6% as the outflow of the node. The production or the attraction of this node is 6%×AADT.

Obviously, if there are multiple links connecting to one node, the authors needed to calculate the total production/attraction by summing over all links. Mathematically, the formula of the gravity model for computing the number of trips between node i and node j is as follows:

$$d_{ij} = P_i \left(\frac{A_j F_{ij} K_{ij}}{\sum_k A_k F_{ik} K_{ik}} \right) \qquad \textbf{(16)}$$

where d_{ij} is the number of trips (demands) from node i to node j, P_i is the number of trip productions in node i, A_j is the number of trip attractions in node j, F_{ij} is the friction factor relating the spatial separation between node i and node j, and K_{ij} is the socioeconomic factor between node i and node j.

There are 70 nodes, 224 links, and 924 OD pairs. The authors assumed that each link had two lanes and each lane had the capacity of 2,000 vehicles per hour. The average speed is assumed to be 60 miles per hour (mph) for the first block of capacity and twice as much as the previous one for the next blocks. The disruption upper bound of links is the capacity of the link which is two lanes and the expansion upper bound is one lane for each link. The upper bound of total disruption in the network in this experiment varies from 5 to 100 lanes for different examples. The expansion budget is assumed to be $800 million and the cost of expansion is estimated to be $1.5 million per lane per mile. The demand of OD pairs is obtained by applying the gravity model.

The authors conducted an experiment to test and compare the performances of the trilevel optimization model and an intuitive expansion strategy. First, the authors assumed that there was no expansion and disruption in the system and estimated the total travel time in the network by solving the lower level problem (Equation 12). Second, the authors ran the bilevel programming problem (middle and lower levels) to assess the resiliency of the network, assuming that there was no expansion, but that probabilistic disruptions could happen through the network. Third, the authors improved the results of the bilevel model by applying intuitive expansions in the network.

After assessing the resiliency, the authors expanded the most congested links, which had the largest ratio of volume over capacity, as much as the expansion budget allowed. After making these expansions, the authors again assessed the resiliency of the network by solving the bilevel problem.

Fourth, the authors ran the trilevel optimization model to confront the worst- case scenario disruptions in the most resilient manner.

When there is no expansion or disruption, the network flow is as shown in Figure 4. The total travel time is 3.6364×10^6 minutes.

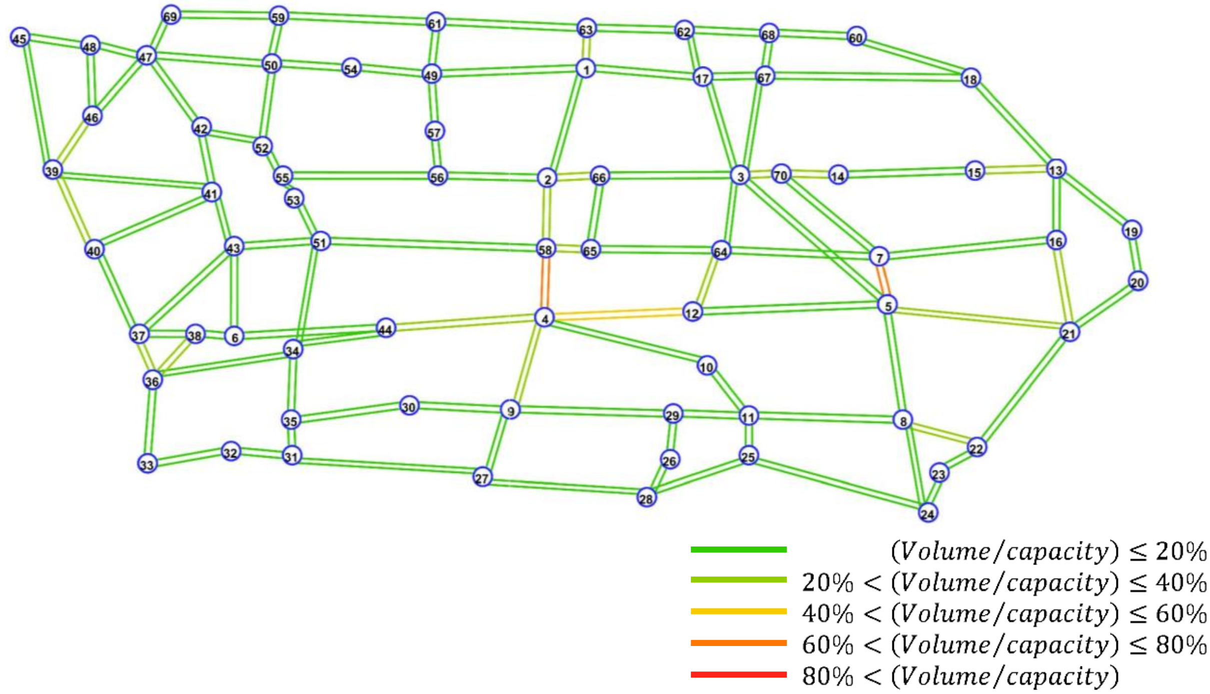


Figure 4. Iowa transportation network without expansion and without disruption

In addition, the authors assessed the resiliency of the network when there is no expansion by solving the bilevel programming problem. There are 448 total links in the network and the authors set the total disruption limit to different values from 5 to 100 lanes. Figure 5 shows the total network travel time with different disruption limits.

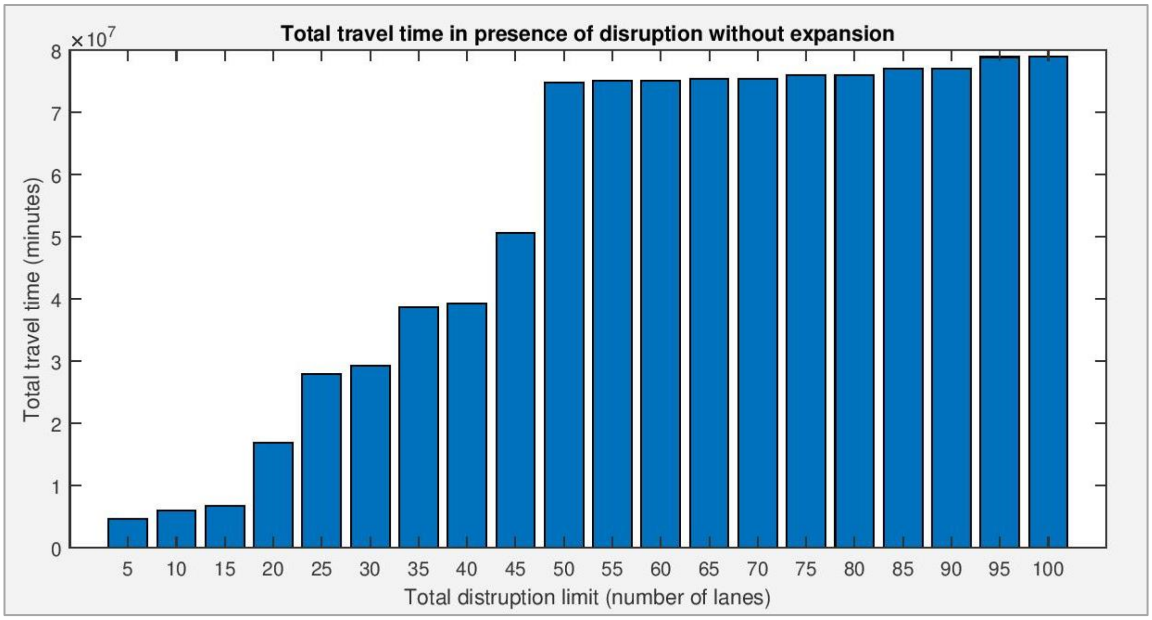


Figure 5. Total travel time in presence of disruption without expansion for different values of disruption limit from 5 to 100 lanes

Figures 6 through 8 depict the Iowa transportation network with disruption limits of 5, 50, and 100 lanes, respectively. The orange and red lines indicate more congested links.

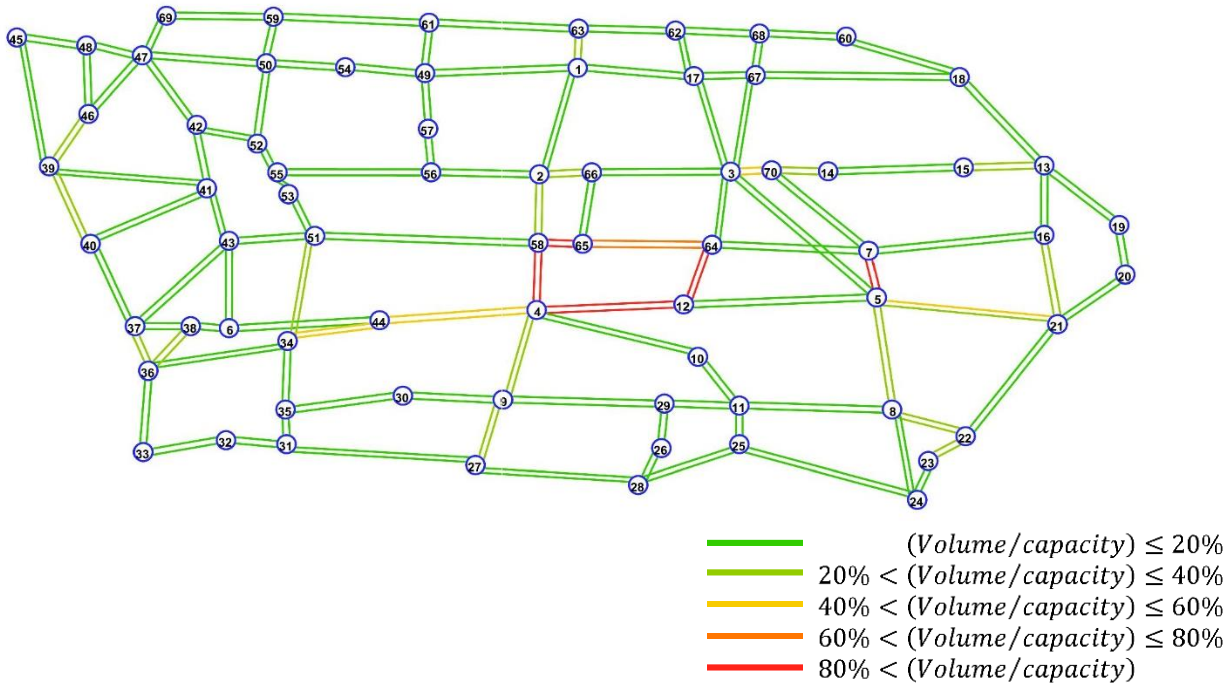


Figure 6. Iowa transportation network when the total disruption is 5 lanes

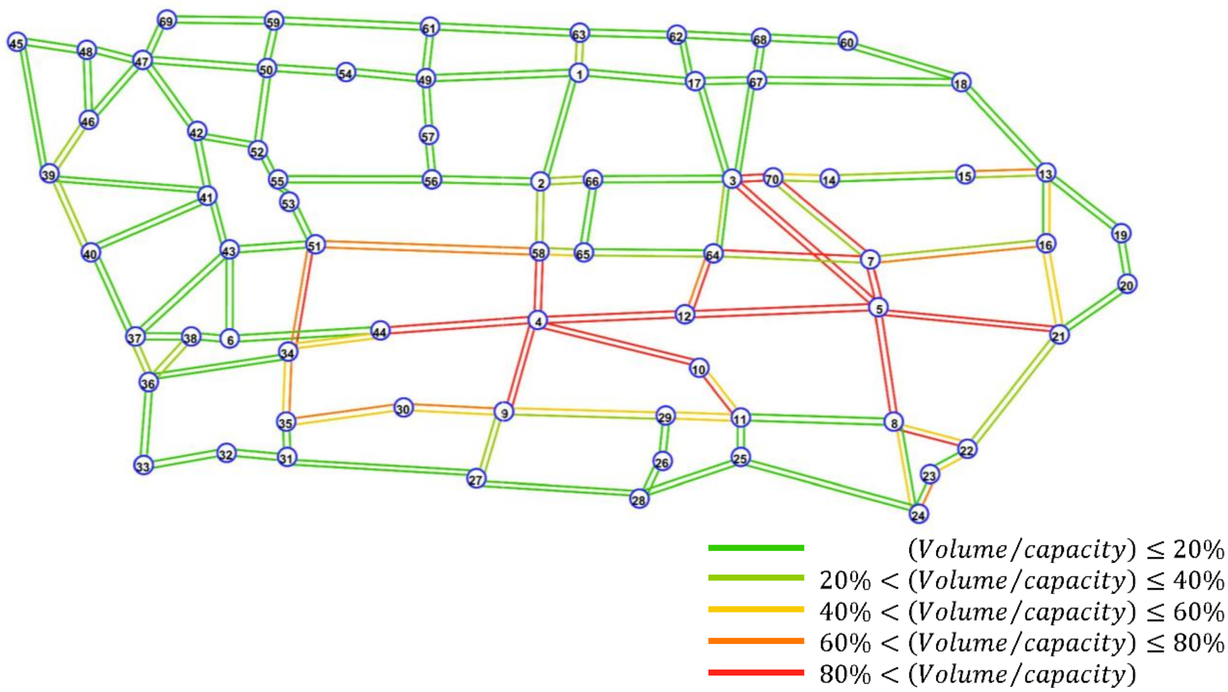


Figure 7. Iowa transportation network when the total disruption is 50 lanes

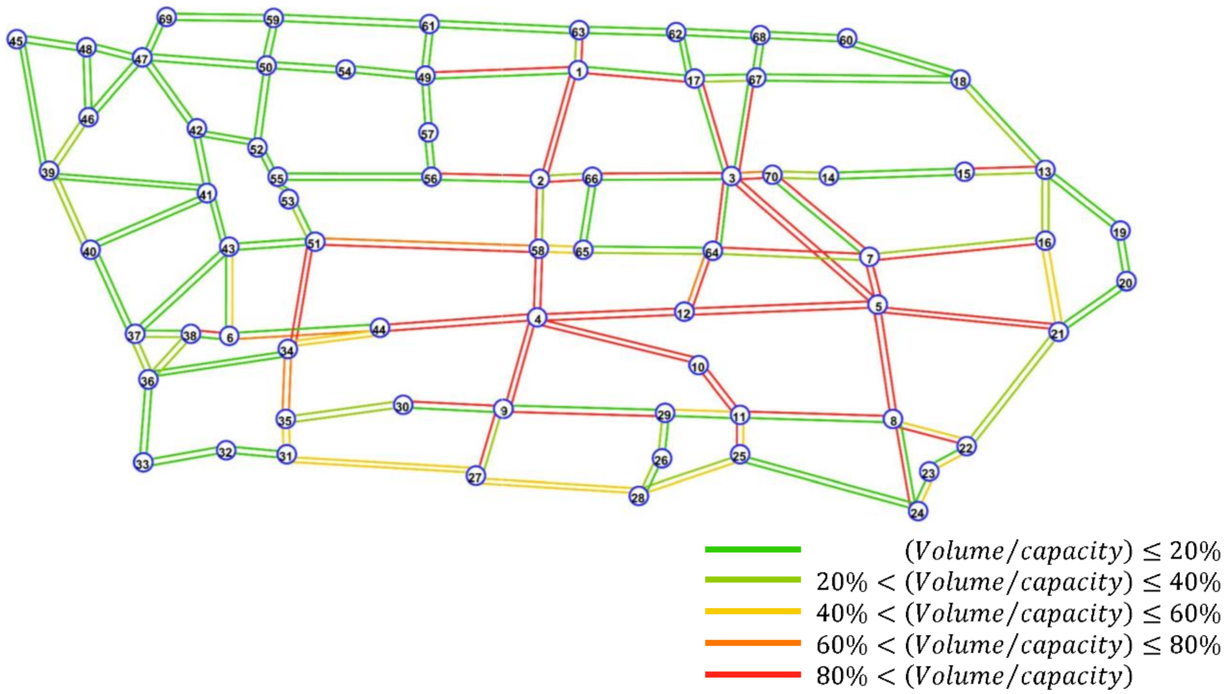


Figure 8. Iowa transportation network when the total disruption is 100 lanes

In the next stage, the authors enhanced the resiliency of the network in two ways: first, by greedily adding lanes to the most congested links; and second, by solving the trilevel optimization model to determine what links needed to be expanded. For the greedy expansion, the authors first solved the bilevel programming problem to assess the resiliency of the network for different disruption limits. Then, the most congested links were expanded as widely as the expansion budget allowed. After expansion, the resiliency of the network was assessed again by solving the bilevel programming problem.

Since the proposed algorithm to solve the subproblem (Equation 15) is a heuristic algorithm, the solution presented for the trilevel optimization model was not an optimal choice. However, the algorithm returned an optimal solution in the case of finding the optimal solution for the subproblem. Figure 9 compares the results of the greedy expansion and the trilevel optimization model. The results show that if more than 10% of the total capacity is disrupted, the trilevel optimization model can enhance the resiliency more than the greedy expansion.

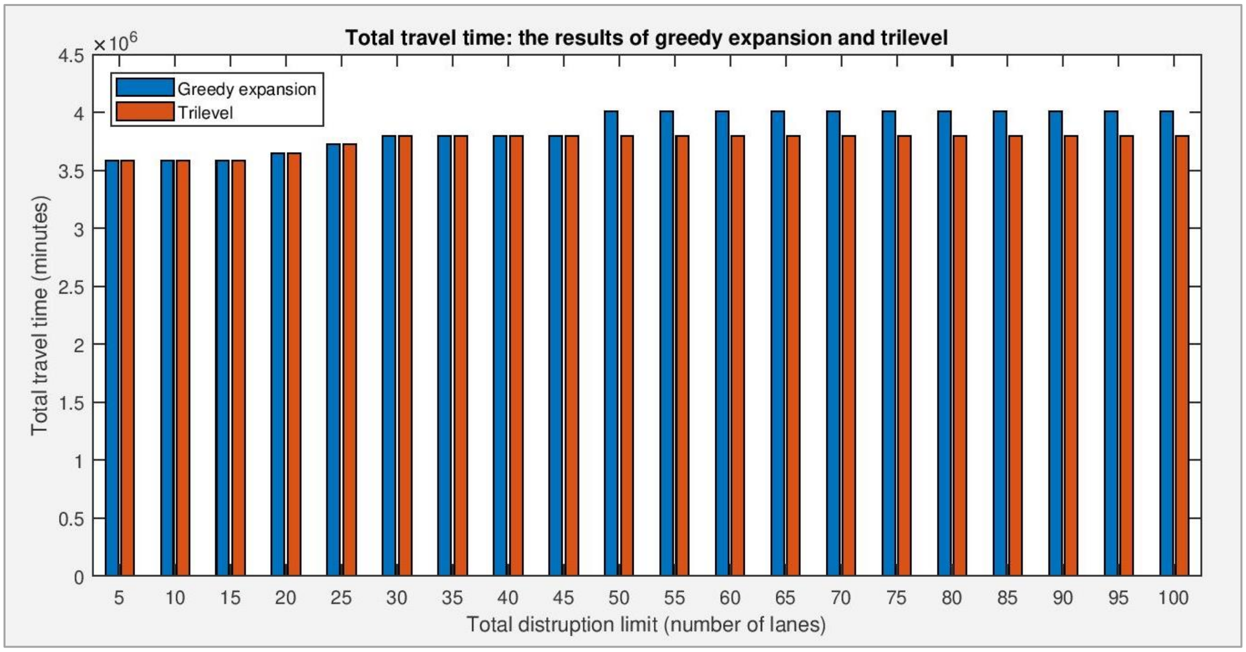


Figure 9. The comparison of total travel times between greedy expansion and the trilevel optimization model for different values of disruption limit from 5 to 100 lanes

CONCLUSIONS

In this study, the authors proposed a new approach to address uncertainty in a transportation network. The link capacities were uncertain parameters and the origin-destination demands were constant. The objective was to design a resilient transportation network to minimize the total travel time in the presence of disruption. This study makes three contributions to the literature.

First, the authors developed a trilevel optimization model for the resilient network design problem as follows:

- The lower level determined the network flow to minimize the total travel time
- The middle level assessed the resiliency of the network by identifying the worst-case scenario disruptions that could lead to maximum cost to the transportation system
- The upper level designed the optimal strategy to expand the existing transportation network so that it enhanced the resiliency of the network

Second, the authors reformulated the network flow problem to significantly reduce the number of variables and constraints.

Third, the authors designed a heuristic algorithm for solving the trilevel optimization model to enhance the resiliency of the network more efficiently.

The results of solving the bilevel programming problem (assuming that there is no expansion) showed that when the upper bound of total disruption in the network is more than 5% of the capacity, the total travel time increased considerably. Therefore, reducing the link capacities due to probabilistic disruptions affects the performance of the transportation network and can delay or stop the movement.

The authors improved the results of the bilevel model by applying two methods. The results showed that the trilevel optimization model outperformed the greedy expansion model.

This study was subject to several limitations that suggested future research directions. For example, the proposed model assumed the origin-destination demands are deterministic. Relaxing this assumption would require a more complicated model that reflects the uncertainty over demands. In addition, the designed algorithm is heuristic. A possible future work could be to design a more efficient algorithm or an exact algorithm to find the optimal solution.

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